

Analysis I (MTH1032)

Worksheet 9

Pre-Workshop Assignment. Understand and memorize

- the definition of a compact set.
- the statement and the main idea of the proof of the theorem that a continuous function on a compact domain attains its minimum and maximum.
- the definitions of differentiability, as well as left and right differentiability.

Part 1: Exercises.

Question 1.

1. Show that every closed and bounded interval $[a, b] \subset \mathbb{R}$ is compact.
2. Show that any non-empty half-open interval $(a, b] \subset \mathbb{R}$ is not compact.
3. Show that any compact set $D \subset \mathbb{R}$ is bounded.

Question 2.

1. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} |x| & \text{if } |x| > 1 \\ x^2 & \text{if } -1 \leq x \leq 1 \text{ and } x \in \mathbb{Q} \\ 0 & \text{else.} \end{cases}$$

Determine the set of points of differentiability

$$D := \{\xi \in \mathbb{R} : f \text{ is differentiable at } \xi\}.$$

Prove your findings.

2. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{\sin \sqrt{|x|}}{\sqrt{|x|}} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0. \end{cases}$$

Is f is right or left differentiable at $\xi = 0$? Is f differentiable at $\xi = 0$?

Part 2: Exam preparation.

Question 1.

What is a compact set? Give examples of sets that are compact and of sets that are not compact.

Question 2. What can be said about the existence of the minimum and maximum of a continuous function on a compact set? Can you prove it? Is that still true without the assumption of compactness? Illustrate your answer by giving examples.

Question 3.

What is the definition of differentiability of a function? Give an example of a differentiable function and a non-differentiable function. Explain the relation of differentiability and continuity.

Question 4.

What is left and right differentiability of a function, and how is it related to differentiability? Can you give an example of a function that is not continuous but left and right differentiable at some point?