

Analysis I (MTH1032)

Worksheet 8

Pre-Workshop Assignment. Understand and memorize

- the Intermediate Value Theorem.
- the definitions of (strict) monotonicity, boundedness, supremum, infimum, maximum and minimum of functions.
- the statement that a strictly monotonic and continuous function has a strictly monotonic and continuous inverse function.

Part 1: Exercises.

Question 1.

Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} |x| & \text{if } |x| > 1 \\ x^2 & \text{if } -1 \leq x \leq 1 \\ 0 & \text{else.} \end{cases} \quad \text{and } x \in \mathbb{Q}$$

1. Attempt to sketch the graph of f .
2. Determine the set of continuity points

$$C := \{\xi \in \mathbb{R} : f \text{ is continuous at } \xi\}.$$

Prove your findings.

Question 2. Find all values for $a, b \in \mathbb{R}$ such that the function $f : \mathbb{R} \rightarrow \mathbb{R}$

$$f(x) = \begin{cases} x^2 + a & \text{if } x \leq 1 \\ bx + 1 & \text{else} \end{cases}$$

is continuous at $\xi = 1$.

Question 3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be two continuous functions such that $f(x) = g(x)$ for all $x \in \mathbb{Q}$. Show that $f(x) = g(x)$ for all $x \in \mathbb{R}$.

Part 2: Exam preparation.

Question 1.

What does the Intermediate Value Theorem say? Draw a picture to illustrate it. Can you give the idea of the proof?

Question 2. What can be said about the image of an interval under a continuous function? How can you prove that?

Question 3.

What is the definition of (strict) monotonicity of a function? What does it have to do with the invertibility of a (continuous) function?

Question 4.

How are the terms supremum, infimum, maximum and minimum for a function defined? Can you give an example of a function that has a finite supremum, no maximum and no finite infimum?