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Analysis I (MTH1032)

Worksheet 7

Pre-Workshop Assignment. Understand and memorize

- the concept of improper limits.
- the concept of continuity.

Part 1: Exercises.

Question 1.

Let $\alpha \in \mathbb{R}$ be a fixed number and let the function $f : (0, \infty) \to \mathbb{R}$ be defined by

$$f(x) = (x+1)^\alpha \ - x^\alpha.$$

Depending on the value of α, determine the limit

$$\lim_{x \to \infty} f(x).$$

Hint: You can distinguish between the cases $\alpha < 0, \alpha = 0, 0 < \alpha < 1, \alpha = 1$ and $\alpha > 1$. For $0 < \alpha < 1$ you may use the inequality $\left(1 + \frac{1}{x}\right)^\alpha \leq 1 + \frac{\alpha}{x}$, $x > 0$, and for $\alpha > 1$ you may use the inequality $\left(1 + \frac{1}{x}\right)^\alpha \geq 1 + \frac{\alpha}{x}$, $x > 0$.

Question 2. Show that the function $f(x) = \cos(x)$, $x \in \mathbb{R}$, does not have a limit at ∞.

Question 3. Show that the function $f : \mathbb{R}\backslash\{-1, 1\} \to \mathbb{R}$, defined by

$$f(x) = \frac{3x^2 + 5x + 5}{x^2 - 1},$$

is continuous at $\xi = -2$ by finding, for every $\epsilon > 0$, a suitable δ as required by the definition of continuity.
Hint: You can use the factorisation $2x^2 + 15x + 22 = (2x + 11)(x + 2)$.

Part 2: Exam preparation.

Question 1.

What does $\lim_{x\to\xi} f(x) = \infty$ mean? Give the definition and a concrete example of such a situation.

Question 2.

What does $\lim_{x\to\xi+} f(x) = \infty$ mean? Give the definition and an example of a function f such that $\lim_{x\to\xi+} f(x) = \infty$ but not $\lim_{x\to\xi} f(x) = \infty$. Can you also find a function f such that $\lim_{x\to\xi} f(x) = \infty$ but not $\lim_{x\to\xi+} f(x) = \infty$?

Question 3.

How is $\lim_{x\to\infty} f(x) = \infty$ defined? Give a concrete example of such a situation.

Question 4.

What is a continuous function? Give the definition and explain the relation to limits. How can we characterise continuity in terms of sequences?