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Analysis I (MTH1032)

Worksheet 4

Pre-Workshop Assignment. Understand and memorize

- the definition of a series
- Cauchy's condensation test
- Dirichlet's test and the main idea of its proof
- the definition of absolute convergence of a series

Part 1: Exercises.

Question 1.

Examine the convergence of the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}}{n+1}.$$

Question 2.

Is the following series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}-(-1)^n}{n}$$

convergent, absolute convergent or divergent?

Question 3.

Compute the sum of the series

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$$

Question 4.

Let $(a_n)_{n=0}^{\infty}$ and $(b_n)_{n=0}^{\infty}$ sequences such that $\sum_{n=0}^{\infty} a_n$ is convergent and (b_n) is bounded and monotonic (increasing or decreasing). Show that the series

$$\sum_{n=0}^{\infty} a_n b_n$$

is convergent.

Part 2: Exam preparation.

Question 1.

State the definition of a series. What does convergence mean in the context of series? Explain the meaning(s) of the symbol $\sum_{n=0}^{\infty} a_n$.

Question 2.

What is Cauchy's condensation test, how does it work? Apply it to the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ in order to show its divergence.

Question 3.

What is Dirichlet's test? Explain roughly the main idea of the proof (without writing down the proof in detail). What happens if we weaken the assumption (b_n) *is a decreasing sequence converging to 0* to simply (b_n) *converges to zero*?

Question 4.

What is absolute convergence of a sequence? How is it related to convergence? Give an example of a series that is convergent and absolutely convergent, and an example of a series that is convergent but not absolutely convergent.