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Analysis I (MTH1032)

Worksheet 3

Pre-Workshop Assignment. Understand and memorize

- the Monotone Convergence Theorem
- the definition of Euler's number e
- the definition of a Cauchy sequence
- the definition of a cluster point of a sequence

Part 1: Exercises.

Question 1.

Let $(a_n)_{n=1}^{\infty}$ be a sequence. A *telescoping sum* is a sum of the form

$$\sum_{k=m}^{n}(a_k - a_{k+1}),$$

where $n, m \in \mathbb{N}$ with $n \geq m$.

1. Show that
$$\sum_{k=m}^{n}(a_k - a_{k+1}) = a_m - a_{n+1}.$$
2. Compute for every $n \in \mathbb{N}$
$$\sum_{k=1}^{n}\left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}\right).$$
3. Compute for every $n \in \mathbb{N}$
$$\sum_{k=1}^{n}\frac{1}{(k+1)(k+2)}.$$

Question 2.

Find the set of cluster points of the sequence $(a_n)_{n=1}^{\infty}$ when

1. $$a_n = (-1)^n + \frac{1}{\sqrt{n}}$$

2. $$a_n = (-1)^n \left(1 + \frac{1}{n}\right)^n$$

Part 2: Exam preparation.

Question 1.

State the Monotone Convergence Theorem. Does the statement hold if we omit the assumption of boundedness or the assumption of monotonicity? Elaborate on your answer.

Question 2.

Explain how Euler's number e is defined. Why is the defining sequence convergent?

Question 3.

State the definition of a Cauchy sequence. What is the connection between Cauchy sequences and convergent sequences?

Question 4.

State the definition of a cluster point of a sequence. Give an example of a sequence and its cluster points. How are cluster points connected to subsequences?