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Analysis I (MTH1032)

Worksheet 2

Pre-Workshop Assignment. Understand and memorize

- the definition of a subsequence of a sequence
- the definitions of (strictly) increasing and (strictly) decreasing sequences
- the proof of the statement that every sequence of real numbers has a monotonic subsequence

Part 1: Exercises.

Question 1.

1. Prove that a subsequence of a convergent sequence always converges to the limit of the sequence.
2. Use this to compute the limits

$$\lim_{n\to\infty} \sqrt[2n+1]{2n+1}, \qquad \lim_{n\to\infty} \frac{n!}{2^{n!}}.$$

Question 2.

1. Let $(a_n)_{n=1}^{\infty}$ be a convergent sequence with limit $a \in \mathbb{R}$. Prove that

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} a_k = a.$$

Hint: Write $\frac{1}{n}\sum_{k=1}^{n} a_k - a = \frac{1}{n}\sum_{k=1}^{n}(a_k - a)$ and split the sum into two parts, each of which you can show to be smaller than $\epsilon/2$.

2. Can you find a divergent sequence $(b_n)_{n=1}^{\infty}$ such that the following limit exists?
$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} b_k$$

3. Can you find a divergent monotonic sequence $(c_n)_{n=1}^{\infty}$ such that the following limit exists?
$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} c_k$$

Part 2: Exam preparation.

Question 1.

What is a subsequence of a sequence of real numbers? Give the formal definition and two examples of a sequence, along with one of its subsequences.

Question 2.

State the definitions of an increasing, strictly increasing, decreasing and strictly decreasing sequence of real numbers. Provide an example for each case.

Question 3.

Explain why every sequence of real numbers has a monotonic subsequence.

Hint: The best way to answer this type of question is to give a proof and explain it!