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Analysis I (MTH1032)

Worksheet 13

Pre-Workshop Assignment. Understand and memorize

- the method of integration by parts.
- the method of change of variables.
- the comparison test for improper Riemann integrals.

Part 1: Exercises.

Question 1.

1. Compute the following integral using the method of integration by parts:

$$\int_0^1 x \log(1+x) dx$$

2. Compute the following integral using the change of variables $1 + x^2 = t$:

$$\int_1^{\sqrt{2}} x\sqrt{1+x^2} dx$$

Question 2.

For some $a > 0$, let $f : [-a, a] \to \mathbb{R}$ be a continuous function. Using a simple change of variables, prove the following statements.

1. If f is an odd function, i.e., $f(-x) = -f(x)$ on $[-a, a]$, then we have

$$\int_{-a}^{a} f(x) dx = 0$$

2. If f is an even function, i.e., $f(-x) = f(x)$ on $[-a, a]$, then we have

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx.$$

Question 3.

Consider the Gamma function $\Gamma : (0, \infty) \to \mathbb{R}$ defined by

$$\Gamma(x) = \int_{0}^{\infty} e^{-t}t^{x-1}dt.$$

Show that the Gamma function interpolates the factorial, i.e., for all $n \in \mathbb{N}$

$$\Gamma(n+1) = n!$$

Hint: Induction and integration by parts.

Part 2: Exam preparation.

Question 1. State the theorem on integration by parts and explain how it works. What is the idea of the proof? Can you apply it to an example?

Question 2. State the theorem on change of variables and explain how it works. How can it be proved? Can you apply it to an example?

Question 3. What is an improper Riemann integral? What is the difference between a proper and an improper integral? Can you give examples?

Question 4. Explain how the comparison test for integrals works and apply it to the Gamma function.