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2025/2026

Analysis I (MTH1032)

Worksheet 12

Pre-Workshop Assignment. Understand and memorize

- the definition of the Riemann integral.
- the concept of a Riemann sum.
- the statement and the main idea of the proof of the Fundamental Theorem of Calculus.

Part 1: Exercises.

Question 1.

Let $f : [a, b] \to \mathbb{R}$ be a piecewise constant function, i.e., there exists a partition $a = x_0 < \ldots < x_n = b$ such that

$$f(x) = \begin{cases} c_i & \text{if } \ x \in (x_{i-1}, x_i), \quad i = 1, \ldots, n, \\ d_i & \text{if } \ x = x_i, \quad i = 0, \ldots, n, \end{cases}$$

where c_i and d_i are fixed real numbers. Explain why $f \in \mathcal{R}([a, b])$ and compute the integral

$$\int_a^b f(x)dx.$$

Hint: You may use Theorems 46 and 47.

Question 2.

For any $a < b$, compute the integral

$$\int_a^b e^x dx$$

using Riemann sums.

Hint: You can choose the partitions P_n given by the points $x_i = a + i\frac{b-a}{n}$ for $i = 0, \ldots, n$, and the sample points $\xi_i = x_i, i = 1, \ldots, n$.

After that, feel frustrated and use the Fundamental Theorem of Calculus to compute the integral in a much easier way!

Part 2: Exam preparation.

Question 1.

What is the definition of a Riemann-integrable function? If it exists, how can the integral of a function be interpreted? Can you give examples of (classes of) functions which are Riemann-integrable? Can you give an example of a (bounded) function that is not Riemann-integrable?

Question 2. What is a Riemann sum, and how is it related to the Riemann integral? Why are these sums useful?

Question 3.

State the Fundamental Theorem of Calculus with all parts, and talk about how can it be proved. Why is it so *fundamental*?