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Analysis I (MTH1032)

Worksheet 11

Pre-Workshop Assignment. Understand and memorize

- the statement of Taylor's theorem.
- the definition of Taylor series and real-analytic functions.
- the Taylor series representations for the logarithm and the binomial series.

Part 1: Exercises.

Question 1.

For each of the following functions, find the Taylor series centered at $x_0 = 0$, compute its radius of convergence, and determine whether the function is real-analytic at $x_0 = 0$.

1. $f : (-\infty, 1) \to \mathbb{R}, \quad f(x) = \frac{1}{1-x}$,
2. $f : (-1, 1) \to \mathbb{R}, \quad f(x) = \frac{1}{1-x^2}$,
3. $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = e^x$,
4. $f : (-1, 1) \to \mathbb{R}, \quad f(x) = \log\left(\frac{1-x}{1+x}\right)$.

Question 2.

For some $n \in \mathbb{N}_0$, let $f : \mathbb{R} \to \mathbb{R}$ be an $(n+1)$-times differentiable function such that $f^{(n+1)}(x) = 0$ on $\mathbb{R}$. Show that f is a polynomial of degree at most n.

Part 2: Exam preparation.

Question 1.

What exactly is the statement of Taylor's theorem? Can you name the idea or the tools that are used in its proof?

Question 2.

What is a Taylor series of a function? When is a function called real-analytic? Can you give an example of a real-analytic function, and an example of a smooth function which is not real-analytic?

Question 3.

Can you name and elaborate on some applications of Taylor's theorem, e.g., specific Taylor series representations or important statements in optimisation?

Question 4.

From a high-level perspective, provide a brief overview of differentiation. What is the main motivation behind the concept, and how does it work? Explain the types of questions that can be answered using differentiation. Can you name some different applications?