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Analysis I (MTH1032)

Worksheet 1

Pre-Workshop Assignment. Understand and memorize

- the definition of a sequence of real numbers
- the definition of convergence of a sequence
- the proof that a sequence has at most one limit
- the definition of boundedness of sequences

Part 1: Exercises.

Question 1.

Using the definition of convergence, prove that the sequence (a_n) defined by $a_n = 1 + \frac{(-1)^n}{n}$, $n \geq 1$, is convergent.

Question 2.

Using the definition of boundedness, prove the following two statements:

If (a_n) and (b_n) are bounded sequences, then the sequences $(a_n + b_n)$ and $(a_n b_n)$ are also bounded.

Question 3.

Consider the sequence (c_k) given by

$$c_k = k^2 - k - 1, \quad k \geq 1.$$

Is this sequence bounded above, bounded below or bounded? Is it convergent? Can you justify your answers by proving them?

Part 2: Exam preparation.

Question 1.

What exactly is a sequence of real numbers? Give the formal definition and three concrete examples of sequences.

Question 2.

Explain the concept of convergence of a sequence and give the formal definition. Give one example of a convergent sequence and one of a divergent sequence.

Question 3.

Can a sequence have more than one limit? Prove your answer!

Question 4.

What is the definition of a sequence bounded below or above, and what is a bounded sequence? Can you give an example of a sequence that is not bounded below?

Question 5.

Is a convergent sequence necessarily bounded? If so, can you prove that? Is the converse true? If not, why not?